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A Closed Dynamic Model to Describe and Calculate the Kondratiev Long Wave of Economic Development

A. A. Akaev and V. A. Sadovnichiy*
Moscow State University, Moscow, Russia
e-mail: askarakaev@mail.ru; p930@rector.msu.ru
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Abstract—A full closed mathematical model to describe and calculate Kondratiev's long wave (LW) of economic development is presented for the first time. The innovative process that generates a new long wave in the economy is described as a stochastic Poisson process. The key role in constructing production functions during both the upward and downward trends of the LW is played by the self-similarity property of the innovative process, which is determined by its fractal structure. The role of the switch from an upward wave to a downward one is played by entrepreneurial profit; this article places primary emphasis on calculation of it. The practical effect of the model developed is illustrated through predictive calculations of GDP movement paths and the number of employees in the economy and the dynamics of fixed physical capital formation and growth of labor productivity by the example of the development of the US economy during the coming sixth Kondratiev LW (2018–2050).

Keywords: Kondratiev cycles, Kondratiev long wave of economic development, US economy, K-waves, K-cycles.

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In the near future, the world scientific community will celebrate the centenary (1922–2022) of the wonderful discovery by the great Russian scientist N.D. Kondratiev, who first detected long waves (LWs) with a period of about half a century in the long-term development of the capitalist economy, which are generated by technological revolutions [1]. Kondratiev's theory of LWs, or major cycles of economic conjunctures (MCCs), rose to international fame owing to publications in European languages [2, 3]. We should certainly note the exceptional role of J. Schumpeter, one of the greatest economists of the 20th century, in promotion of Kondratiev's theory. He was enthusiastic about the doctrine of MCCs and developed an innovative LW theory, integrating it into his general theory of economic development [4]. He was also the first to propose to call LWs after Kondratiev; since then, the scientific literature has called them K-waves in the development of the capitalist economy. Emphasizing the extreme topicality of LW theory for analysis of the modern economy, M. Hirooka called it Kondratiev's epoch-making discovery [5]. Proceeding from Schumpeter's innovative theory of economic development and Kondratiev's LW theory, Hirooka worked out a consistent theory of technological innovation in the modern capitalist economy. He was also the first to form an empirical proof of the important thesis that the diffusion of basic innovations is fully synchronized with the upward trend of MCCs and reaches saturation in the region of the highest peak of a cycle. However, there is no closed mathematical model thus far that would describe the dynamics of the main economic indicators: changes in the GDP and the number of employees, the accumulation of fixed physical capital, and increase in labor productivity. On the eve of the 125th anniversary of the birth of Kondratiev, we decided to fill this gap and have developed a closed dynamic model of MCCs, which includes an endogenous mechanism of LW development.

MODELS TO DESCRIBE LONG-TERM TREND PATHS OF ECONOMIC DEVELOPMENT

Let us single out two components in the economy of the country under study, one of which is traditional life-supporting industries that determine the long-term trend of economic development and the other is new industries and sectors generated as a result of the assimilation of innovative technologies and products. For the sake of future analysis, we chose the economy of the United States (Fig. 1) and found out that, for the period from 1950 through 2015, it is ideally described

* RAS Foreign Member Askar Akaevich Akaev, Dr. Sci. (Eng.), is a Professor at Moscow State University. RAS Academician Viktor Antonovich Sadovnichiy is Rector of Moscow State University.

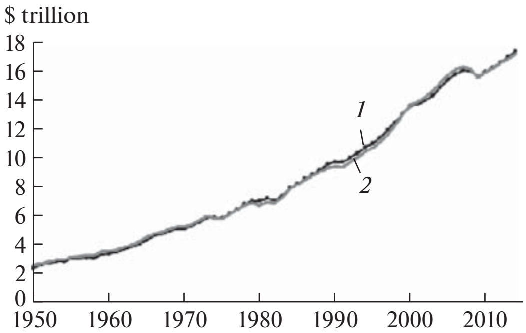

Fig. 1. Paths of growth of (*1*) the actual and (*2*) the estimated GDP of the United States for the period from 1950 through 2015.

by the classical production function with Hicks-neutral technical progress [6]:

$$Y_f = A_f K_f^{\alpha} L_f^{1-\alpha}, \quad \alpha = 0.48, \qquad (1)$$

where Y_f is the GDP, A_f is technical progress, K_f is fixed physical capital, L_f is the number of employees in the economy, and α is a parameter.

Formula (1) presents long-term trend paths of changes in all main macroeconomic variables (marked by the index f). Considering that everywhere (in both developed and developing economies) technological advance has tended to slow down in recent decades, which, in turn, has caused a slowdown in economic growth, we use logistic functions of the following type in order to describe and calculate the trend paths of all main variables:

$$Y_f(t) = \frac{Y_{fm}}{1 + (y_{fmo} - 1)\exp[-\mu_{Y_f}(t - T_0)]}, \quad y_{fmo} = \frac{Y_{fm}}{Y_{fo}}, \qquad (2)$$

where $Y_f(t)$ is the function that describes the trend path of GDP dynamics, Y_{fm} is the maximum GDP value reachable under this technological mode, Y_{fo} is the GDP value at the reference time ($T_0 = 1950$) for the long-term series of economic variables used (United States, 1950–2015), and μ_{Y_f} is the parameter characterizing the rate of change in trend values of the GDP. The numerical values of the constant parameters μ_{Yf} and Y_{fm} in formula (2) were assessed by the least square method using a series of factual data $Y(t)$ for the period from 1950 through 2015.

The trend paths of $A_f(t)$, $K_f(t)$, and $L_f(t)$ were determined similarly. Of course, the logistic trend path is only for countries that ensure long-term sustainable development. Hence, for countries with unstable socioeconomic dynamics, it is necessary to find in each case an adequate approximating function different from the logistic one (2).

Henceforth, we will assume that the accumulation of fixed physical capital (K) and the formation of gross investments (I) in fixed capital are described by the classical equations [6]:

$$\dot{K} = I - \delta K, \quad I = sY, \qquad (3)$$

where δ is the rate of fixed capital retirement and s is the rate of accumulation. For the US economy, we determined the mean values of the above parameters for the period from 1995 through 2015: $\delta = 0.56$; $s = 0.17$. In addition, we established earlier [7, p. 32] that, in the long term, the Kaldor empirical law still holds for the US economy [8]:

$$Y \cong kK, \quad k = \text{const}. \qquad (4)$$

The average value of this parameter for the period from 1985 through 2015 turned out to be $k = 0.32$.

We consider the most general case of economic dynamics, although we could identify significant periods of balanced growth, when the rates of increase in the main economic variables are constant and the following additionally takes place:

$$\frac{\dot{K}}{K} = \frac{\dot{Y}}{Y} = q_Y = \text{const}, \qquad (5)$$

which follows directly from (4). It is natural that equations (3) and (4), as well as relations (5), most accurately describe the values of the respective variables and, hence, are also true for economic variables with the index f.

MATHEMATICAL MODELS TO DESCRIBE AND CALCULATE THE PATHS OF DEVELOPMENT OF NEW ECONOMIC INDUSTRIES AND SECTORS

The upward trend. Let us consider separately upward and downward MCC trends. Suppose that a cluster of innovative technologies has already been formed and the production of innovative goods has begun. Furthermore, these goods, in turn, have actively begun to break into markets and have already covered several percent of their potential volume. Indeed, in 2014–2015, the economies of a number of developed countries (the United States, Germany, Japan, and others) began to demonstrate a pickup, and, starting from 2017–2018, the sixth LW is expected to rise, which is determined by the powerful action of convergent NBIC technologies. In the economies of the most developed countries, i.e., the United States, Japan, and Germany, the market of innovative NBIC-based goods and services has already reached 5–7%. Moreover, the period from 2014 through 2020 is the most favorable for the assimilation and expansion of the new wave of basic NBIC-based innovations [7, p. 95–97]. Since, by the beginning of the large-scale distribution of innovative products to markets, their principal basic models have already been developed, the further process of improving the functional properties and the quality of the products in the upward MCC trend develops exclusively at the expense of enhancing technological innovations [9, 10]. Indeed, in periods of a favorable conjuncture, entrepreneurs limit themselves to innovations that require low-risk investments.

During a depression MCC phase, when the very existence of a great number of business firms is endangered by bankruptcy, the most sagacious entrepreneurs undertake high risk associated with the production of basic innovative products and changes in technologies, because they understand that even a significant modernization of old products will not improve the financial situation cardinally. As was established by Hirooka, the path of the distribution of new products to markets is fully synchronized with the upward MCC wave and is of a fractional character [5, p. 297–313]. The latter circumstance determines the self-similar property of the evolution of the market of innovative products [11] and the distribution of its potential, corresponding to the Pareto principle. Let us add to all the above the key idea about the growth of labor productivity in the learning-by-doing process, which was formulated by Nobel Prizewinner K.J. Arrow [12]. The above three principles make it possible to determine in full the functional description and calculations of the paths of the economic development of new industries in the upward MCC trend.

Technological advance. Let us denote the level of technological advance in new economic industries by $a(t)$. Technological advance in the upward MCC trend comes down exclusively to the series of enhanced technological innovations, which are described well by the stochastic Poisson process with the Erlang density distribution function of order k [11]:

$$\lambda_k(t) = \frac{\lambda_0(\lambda_0 t)^{k-1}}{(k-1)!} e^{-\lambda_0 t}, \tag{6}$$

where $\lambda_0 = \text{const}$; $k = 1, 2, \ldots$.

We consider it expedient to limit ourselves to the Erlang function of the third order ($k = 3$):

$$\lambda(t) = \lambda_3(t) = \frac{\lambda_0^3}{2} t^2 e^{-\lambda_0 t}. \tag{7}$$

This function reaches its maximum at the point

$$t_m = \frac{2}{\lambda_0}. \tag{7a}$$

The expected value (σ) of the Poisson process, which describes the dynamics of the improvement of the functional properties and the qualities of innovative products, can easily be calculated by the following formula:

$$\sigma(t) = \int_0^t \lambda_3(\tau) d\tau = 1 - \frac{1}{2}\left[(\lambda_0 t)^2 + 2\lambda_0 t + 2\right] e^{-\lambda_0 t}. \tag{8}$$

Here the question arises about the choice of a suitable value of the parameter λ_0. It follows from (8) that, with $t \to \infty$, $\sigma(t) \to 1$. In practice, $\sigma_{\max}$ is limited to the level $1 - \eta$, η, as a rule, being chosen as 0.1. Consequently, from (8) we obtain the following equation to determine time $T_{\max}$, under which $\sigma_{\max} = 1 - \eta$ is reached:

$$(\lambda_0 T_{\max})^2 + 2\lambda_0 T_{\max} + 2 = 2\eta e^{\lambda_0 T_{\max}}. \tag{9}$$

The solution of this equation is

$$\lambda_0 T_{\max} = 5.3 \quad \text{under} \quad \eta = 0.1. \tag{9a}$$

Since the diffusion of innovative products lasts 20–25 years [5], a suitable value for the parameter λ_0 is

$$\lambda_0 = \frac{1}{4} = 0.25. \tag{9b}$$

Note that $T_{\max} \cong 21.2$years, and, according to (7a), the time of the maximal intensity of the diffusion occurs in $t_m = 8$ years. This is the time after which the "storm of innovations" [13] begins to attenuate. Of course, one can pose and solve the task of the optimal choice of values of the λ_0 parameter; in this case, however, the priority is to describe the dynamics of the long wave of economic development. This is why hereafter we will limit ourselves only to parameter estimators.

The technological level in innovative industries of the economy $a(t)$ is connected with the average technological level across the economy $A(t)$ through the Dubovskii equation [14, p. 5]:

$$\frac{\dot{A}}{A} = \frac{I}{K}\left(\frac{a}{A} - 1\right). \tag{10}$$

It is noted that the values $\frac{a}{A}$ vary from 1.05 to 1.65 [14, p. 7]. Since $I = sY$ (3) and $K = k^{-1}Y$ (4), then

$$a(t) = A(t) + \frac{1}{sk}\dot{A}(t). \tag{11}$$

Thus, the average level of technological advance across the entire economy is determined by the rapidly growing technological level in innovative industries (10). This was first established by the British economist C. Freeman [15]. He states in this respect that a long-term rise is not a result of technical innovations in one or more industries but rather a consequence of the diffusion of the new technical–economic paradigm from several leading sectors into the rest of the economy and particularly into industries that produce capital goods and respective services [16, p. 209].

Actually, $A(t)$ is the trend of technological advance within an individual MCC, while A_f in (1) is the secular trend. As is known [4, 5], $A(t)$ can be described by the logistic function, which we will take in the following form:

$$A(t) = A'_{\text{в}} + \frac{\Delta A c}{1 + (c-1)\exp[-\mu_A(t - T_{\text{в}})]}, \quad (12)$$

where c is the number of levels of the discreteness of the logistic function (usually it is assumed that $c = 10$ or 20), $\Delta A = \frac{A_e - A_{\text{в}}}{c-2}$, A_e and $A_{\text{в}}$ are the final and initial values, $A'_{\text{в}} = A_{\text{в}} - \Delta A = \frac{A_{\text{в}}(c-1) - A_e}{c-2}$, μ_A is the parameter that characterizes the growth rate of the logistic function, and T_e and $T_{\text{в}}$ are the years that correspond to the final (A_e) and initial ($A_{\text{в}}$) values of the technological level. It follows from Eqs. (11) and (12) that, if $t \to +\infty$, $a(t) \to A'_e = A_e + \Delta A$, while with $t \to -\infty$, $a(t) \to A'_{\text{в}} = A_{\text{в}} - \Delta A$. Consequently, we can assume that $\frac{A_e}{A_{\text{в}}} \cong \frac{1.65}{1.05} \cong 1.6$. Since the economy of the United States passed the bottom of the current depression in 2014 and began to rise, we can be certain that this is the beginning of the sixth MCC; i.e., $T_{\text{в}} = 2014$. Then, from the trend path of the development of technological advance $A_f(t)$, we determine $A_{\text{в}} = A_f(t)\big|_{t=T_{\text{в}}} = 0.173$. We previously evaluated the duration of the sixth MCC in paper [17]: $T_e - T_{\text{в}} \cong 36$ years; hence, $T_e = 2050$.

In order to determine the parameter μ_A, let us use the symmetrical position of the point of inflection (12):

$$T_{\text{в}e} = \frac{T_{\text{в}} + T_e}{2},$$
$$\frac{A_{\text{в}} + A_e}{2} = A'_{\text{в}} + \frac{\Delta A c}{1 + (c-1)\exp[-\mu_A(T_{\text{в}e} - T_{\text{в}})]}. \quad (12a)$$

Plugging the expressions for $A'_{\text{в}}$, ΔA, and $T_{\text{в}e}$ into the latter equation and solving it, we obtain

$$\mu_A = \frac{2}{T_e - T_{\text{в}}}\ln(c-1). \quad (12b)$$

From this follows the assessment $\mu_A = 0.137$.

Thus, we have determined all the parameters comprising logistic function (12). Let us write it in a form convenient for further use:

$$A(t) = \frac{A_{\text{в}}(c-1) - A_e}{c-2} + \frac{(A_e - A_{\text{в}})\frac{c}{c-2}}{1 + (c-1)\exp[-\mu_A(t - T_{\text{в}})]}. \quad (13)$$

Now we easily find the derivative:

$$\dot{A}(t) = \frac{(A_e - A_{\text{в}})\frac{c(c-1)}{c-2}\mu_A\exp[-\mu_A(t - T_{\text{в}})]}{\{1 + (c-1)\exp[-\mu_A(t - T_{\text{в}})]\}^2}. \quad (14)$$

The maximal value of this derivative occurs just at the point of inflection ($t = T_{\text{в}e}$):

$$\dot{A}_m = \frac{1}{4}(A_e - A_{\text{в}})\frac{c\mu_A}{c-2}. \quad (14a)$$

Let us write basic equation (11) in the following form:

$$a(t) = A(t) + \varepsilon \cdot \frac{\dot{A}(t)}{\dot{A}_m}, \quad \text{where} \quad \varepsilon = \frac{\dot{A}_m}{sk}. \quad (15)$$

It turns out that $\varepsilon = 0.09$. If we approximate $A(t)$ in this equation by logistic function (13) and replace $\frac{\dot{A}(t)}{\dot{A}_m}$ by $\sigma(t)$ (8), which is similar in nature, we will obtain an approximate equation for predictive calculations of $a(t)$ dynamics:

$$a(t) = A(t) + \varepsilon\sigma(t). \quad (16)$$

In addition, this equation will make it possible to account for the asymmetry of the upward and MCC downward trends through a separate description of $\sigma(t)$ at the respective stages. The growth path of $A(t)$ forms under the influence of autonomous investments, which are prepared primarily by revolutionary changes in equipment and technologies and the innovative products and production processes that they generate. It is autonomous investments that give the first impetus to the upward movement of the economy and determine its trend. Using a multiplier, they have a multiplier effect on the income and then, through stimulated demand and induced investments, bring it to levels characteristic of a boom. They also include investments that compensate for fixed capital retirement. As for induced investments, they are a result of the increase in final demand or sales of innovative goods. They generate the acceleration mechanism that is inserted in the addend of (16), determined by the growth of sales of innovative goods and services, which is described by the function $\sigma(t)$ (8). Thus, the multiplier–accelerator interaction mechanism yields cumulative self-sustained economic growth. When autonomous investments become depleted, induced investments also decrease [18, p. 402], which leads to the exhaustion of the source of further growth.

Next, we can calculate the growth rates of labor productivity in new industries:

$$q_a = \frac{\dot{a}}{a} = \frac{\dot{A}(t) + \varepsilon\dot{\sigma}(t)}{A(t) + \varepsilon\sigma(t)}. \quad (17)$$

Figure 2 shows graphs of $a(t)$ and $q(t)$, calculated under various values of the parameter

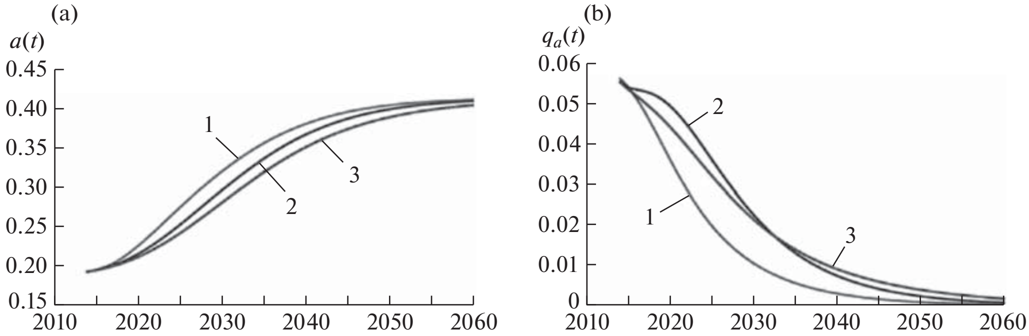

Fig. 2. Dynamics of (a) the increase in the technological level and the dynamics of (b) the slowdown rates of labor productivity in innovative industries of the economy.

$\lambda_0\left(\lambda_0^{(1)}=\frac{1}{4};\ \lambda_0^{(2)}=\frac{1}{6};\ \lambda_0^{(3)}=\frac{1}{8}\right)$. It is clear from Fig. 2b that in the 2030s the growth rates of productivity in innovative industries of the economy will decrease to 1%, which is characteristic of traditional industries. On the other hand, taking into account that the growth of productivity, which is determined by the level of technological advance $a(t)$, fully depends on learning by doing in the workplace, we can use Arrow's model [7, p. 18]:

$$a(t)=\nu\left(\frac{K_{i\text{в}}}{L_{i\text{в}}}\right)^{\theta}, \tag{18}$$

where θ is the Arrow parameter ($\theta\le 1$), ν is the proportionality coefficient, and the index iв means that the basic variable belongs to innovative industries of the economy. Arrow established that, for the aviation industry, $\theta\cong 0.7$. D. Sahal maintains that, in many traditional industries, $\theta\cong 0.32$ [11].

The dynamics of the accumulation of capital and innovative product output. Since $a(t)$ is known (16), formula (18) makes it possible to express $K_{i\text{в}}$ through $L_{i\text{в}}$:

$$K_{i\text{в}}=\left(\frac{a}{\nu}\right)^{\frac{1}{\theta}}L_{i\text{в}}. \tag{19}$$

Let us take the logarithmic derivative:

$$\frac{\dot{K}_{i\text{в}}}{K_{i\text{в}}}=\frac{\dot{L}_{i\text{в}}}{L_{i\text{в}}}+\frac{1}{\theta}\frac{\dot{a}}{a}. \tag{20}$$

The obtained rate equation means that the accumulation rates of innovative physical capital are determined by the growth of technological advance and labor in new economic industries.

The output ($Y_{i\text{в}}$) in new economic industries and sectors, united by a cluster of innovative technologies, is described by a production function (PF) with Solow-neutral technical progress [6], because the upward trend primarily uses capital-saving technologies:

$$Y_{i\text{в}}=(aK_{i\text{в}})^{\beta}L_{i\text{в}}^{1-\beta}. \tag{21}$$

As was mentioned above, the dynamics of production output in an upward MCC trend obeys the self-similarity principle. In determining the self-similarity property, dimensional analysis plays an important role [20, p. 38]. The dimensions of the control variables $Y_{i\text{в}}$, $K_{i\text{в}}$, $L_{i\text{в}}$, and $a(t)$, which are part of PF (21), are as follows:

$$[Y_{i\text{в}}]=P,\quad [K_{i\text{в}}]=PT,\quad [L_{i\text{в}}]=L,\quad [a(t)]=\frac{1}{T}, \tag{22}$$

where P is the product price, T is time in years, and L is the number of workers.

The dimensionalities of all control variables are 1
independent; hence, according to the π theorem [20, p. 41], the dimensionless parameter π, which corresponds to the variable under determination, $Y_{i\text{в}}$, is a constant and has the following form:

$$\Pi=\frac{Y_{i\text{в}}}{K_{i\text{в}}aL_{i\text{в}}^{0}}=c_Y=\text{const}. \tag{23}$$

The constancy of the function π means that output evolves in a self-similar manner. It follows directly from this that

$$Y_{i\text{в}}=c_Y aK_{i\text{в}}. \tag{24}$$

Thus, in PF (21), $\beta=1$, and we arrive at the Lucas AK model of endogenous growth with a constant return on factors of production [21]. The absence of a dependence of output on labor input in PF (24) is compensated by the inclusion in the fixed capital $K_{i\text{в}}$ of not only physical but also human capital, expressed by the result of the increase in production return in the course of learning by doing, which is contained in Arrow's model (18). It is easy to show that the equilibrium in the AK model is Pareto optimal [19]. Consequently, transferring to a constant return on factors of production does not import any sources of market failures. Instead, transferring to the downward MCC trend and labor-saving technologies will again trigger diminishing returns on factors of production.

It follows from relations (19) and (24) that, if we define the functional expression by $L_{i\text{в}}$ or $K_{i\text{в}}$, we can calculate the paths of development of all main economic variables. This will require describing the dynamics of the growth of profit (P), because it is profit that is the main stimulus for economic activity in a market economy. Profit is one of the most important regulators of investment activity. For example, neoclassical theory proceeds from the assumption that the main factor to determine the intensity of the investment process is current profit P(t) [22, p. 76]. In addition, the change in the proportion between capital and labor (19) that takes place in the course of cyclic fluctuations forms a substantial element of neoclassical cycle theory [22, p. 77]. As we see from relation (19), this proportion is determined by $a(t)$. Schumpeter maintained that it is the dynamics of the rate of profit

$$\rho(t) = \frac{\mathrm{P}(t)}{PK(t)} \qquad (25)$$

that determines variations in the intensity of innovations and transfer to new products and new production technologies [13].

The dynamics of the rate of profit in innovative economic industries. Economic firms use capital (K) to make profit (P). Hence, the value of the optimal size of capital is determined by each firm from the condition of maximal profit [19]:

$$\mathrm{P} = PQ - (wL + RK), \quad Q = (AK)^{\alpha} L^{1-\alpha}, \qquad (26)$$

where Q is the volume of real output; P is the overall price level in the economy; w is the nominal wage rate (the real wage being $\overline{w} = \frac{w}{P}$); R is the rental price of capital, $\frac{R}{P} = r + \delta$; r is the interest rate; and δ is the depreciation rate. The formula to determine the value of real profit can be written as follows:

$$\overline{\mathrm{P}} = \frac{\mathrm{P}}{P} = Q - \overline{w}L - (r + \delta)K. \qquad (26a)$$

With respect to our case (in innovative economic
2 industries in the upward MCC trend), we can concretize formula (26a) to calculate profit by plugging concrete expressions for $Q = Y_{i\text{в}}$ (24) and $K_{i\text{в}}$ (19) into it:

$$\overline{\mathrm{P}}_{i\text{в}} = \left\{ \left(\frac{a}{\nu}\right)^{\frac{1}{\theta}} [c_Y a - (r + \delta_{i\text{в}})] - \overline{w} \right\} L_{i\text{в}}, \qquad (27)$$

where $\delta_{i\text{в}}$ is the replacement rate in innovative industries of the economy. There are sources (for example, [23]) that hold that characteristic of high-tech science-intensive industries and sectors similar to the incipient NBIC technologies is a depreciation rate of 12–18%, on average. This relates to computer production and microelectronics, medical instrument making and the pharmaceutical industry, and the production of equipment for R&D. For calculations, we took the average value from the above range:

$$\delta_{i\text{в}} = 0.15 \ (15\%). \qquad (27a)$$

The interest rate $r = 0.08$ (8%) is characteristic of the US economy as a whole.

Since the real wage rate, which maximizes profit, is determined from the condition [6]

$$\overline{w}_{i\text{в}} = \frac{\partial Y_{i\text{в}}}{\partial L_{i\text{в}}}, \qquad (28)$$

it is necessary to obtain the function $Y_{i\text{в}} = f(L_{i\text{в}})$. This function follows directly from relations (24) and (19):

$$Y_{i\text{в}} = c_Y a K_{i\text{в}} = c_Y a \left(\frac{a}{\nu}\right)^{\frac{1}{\theta}} \cdot_{i\text{в}}. \qquad (28a)$$

It follows thence that

$$\overline{w}_{i\text{в}} = c_Y a \left(\frac{a}{\nu}\right)^{\frac{1}{\theta}}. \qquad (28b)$$

In practice, firms long ago ceased to pay optimal-value wages (28b). Real wages are much lower and are determined in the following way:

$$\overline{w}^*_{i\text{в}} = \frac{c_Y}{1+m} a \left(\frac{a}{\nu}\right)^{\frac{1}{\theta}}, \qquad (28c)$$

where m is a monopoly addition to price costs. Currently, the value characteristic of the United States is $m = 0.5$–0.8 [24]. In our forecast calculations, we assumed the lower value $m = 0.5$, supposing that, in the decades to come, capitalists will have to increase the share of wages in income to avoid social conflicts caused by the sharply increased level of inequality.

Plugging the real $\overline{w}^*$ (28c) instead of the optimal $\overline{w}$ (28b) into (27), we obtain a formula to calculate the dynamics of the growth of real profit for entrepreneurs:

$$\begin{aligned} \overline{\mathrm{P}}^*_{i\text{в}} &= \left(\frac{a}{\nu}\right)^{\frac{1}{\theta}} \left[\frac{c_Y m}{1+m} a - (r + \delta_{i\text{в}})\right] L_{i\text{в}} = \\ &= \left[\frac{c_Y m}{1+m} a - (r + \delta_{i\text{в}})\right] K_{i\text{в}}. \end{aligned} \qquad (29)$$

A rather simple formula to calculate the real profit rate follows from this (25):

$$\rho^*_{i\text{в}} = \frac{c_Y m}{1+m} a - (r + \delta_{i\text{в}}). \qquad (30)$$

As was expected, the profit rate grows as $a(t)$ labor productivity increases. Let us introduce a biased rate of profit:

$$\rho^{**}_{i\text{в}} = \rho^{*}_{i\text{в}} + (r + \delta_{i\text{в}}) = \frac{c_Y m}{1+m} a. \tag{30a}$$

The growth rates of the biased rate of profit are

$$q_{\rho^{**}_{i\text{в}}} = \frac{\dot{\rho}^{**}_{i\text{в}}}{\rho^{**}_{i\text{в}}} = \frac{\dot{a}}{a} = q_a. \tag{30b}$$

We see that they are dropping and are just the same as the slowdown rates of labor productivity (see Fig. 2b); by the end of the upward MCC trend, they practically reach zero, which is a signal for entrepreneurs that the downward MCC trend is beginning. Thus, it is profit that preconditions the upper "pivotal point," i.e., the cessation of growth and the transfer from the upward trend to the downward one. Therefore, the rate of profit is the most important component of the LW model, which ensures the endogeneity of economic development in the course of a major cycle.

The investment process equation. As was mentioned above, current profit is the main factor that determines the intensity of the investment process. In forming the investment function, early models, for example, Rose's model, confined themselves to the prerequisite that the higher the rate of profit, the higher the growth rates of capital $q_K = \frac{\dot{K}}{K}$ are, i.e., to the condition $\frac{\partial q_K}{\partial \rho} > 0$ [22, p. 85]. However, the functional dependence $q_K = f(\rho)$ itself was not disclosed. We assume the hypothesis that

$$q_K = \frac{\dot{K}}{K} = \frac{I(t)}{K(t)} \sim \dot{\rho}(t). \tag{31}$$

The right-hand side of the capital formation equation lacks the depreciation term because it is part of autonomous investments. Hence, only pure induced investments $I(t)$ are present in the equation. The classical monograph by A. Hansen [18] shows that the maximal volume of pure investments is reached at the inflection point of the capital accumulation curve. As for the latter, it resembles $a(t)$, according to formula (18). Hence, their inflection points are close to one another; then, since the profit rate (30) is fully determined by technological advance $a(t)$, it is obvious that $I_{i\text{в}}/K_{I\text{в}} \sim \dot{\rho}_{i\text{в}}$. However, this is just only in the upward trend, while the downward one demonstrates the simple relation $I \sim \mathrm{P}$, i.e., according to the principle "the smaller the profit, the fewer the investments."

Derivation of formulas to calculate the main indicators of innovative industries of the economy. Taking into consideration rate equation (20) and relation (30), we can write equation (31) for pure investments in the following form:

$$\frac{\dot{L}_{i\text{в}}}{L_{i\text{в}}} = \frac{v c_Y m}{1+m}\dot{a} - \frac{1}{\theta}\frac{\dot{a}}{a}, \tag{32}$$

where v is the coefficient representing the share of profit that entrepreneurs allocate to a real pure investment program. Publication [25] presents the results of empirical studies for the economy of Great Britain, which show that, in the upward trend of the fifth MCC (1998–2000), $v \cong 0.55$, while in the downward trend (2004–2008), $v_r \cong 0.42$. Although we have not found respective data for the US economy, taking into account the similarity of the behavior of American and British entrepreneurs, we assume the above data for predictive calculations for the period of the sixth MCC.

Differential equation (32) has a rather simple solution:

$$\bar{L}_{i\text{в}} = \frac{L_{i\text{в}}}{L_{i\text{в}6}} = \exp\left\{\frac{v c_Y m}{1+m}\left[a(t) - A_6\right] - \frac{1}{\theta}\ln\left[\frac{a(t)}{A_6}\right]\right\}. \tag{33}$$

$L_{i\text{в}6}$ here is initial labor already employed in new industries of the economy by the beginning of the sixth MCC, i.e., by the moment $t = T_6 = 2014$; A_6 is the average technological level in the economy by the beginning of the sixth MCC, when $a_6 = A_6$. Having specified the values $A_6 = A_\text{в} = 0.173$, $v = 0.55$, $m = 0.5$, and $\theta = 0.7$, we determined from Eq. (33) by the least squares method $c_Y = 105$ proceeding from the two known values of $L_{i\text{в}}$: in 2008, $L_{i\text{в}} = 0.15$ mln people; in 2015, $L_{i\text{в}} = 0.8$ mln people [26].

Plugging the functional expression of $\bar{L}_{i\text{в}}$ (33) into relations (19), (24), and (29), we find explicit expressions for other main economic variables:

$$\bar{K}_{i\text{в}} = \frac{K_{i\text{в}}}{K_{i\text{в}6}} = k_{i\text{в}}\left(\frac{a}{A_6}\right)^{\frac{1}{\theta}} L_{i\text{в}}, \tag{34a}$$

$$\bar{Y}_{i\text{в}} = \frac{Y_{i\text{в}}}{Y_{i\text{в}6}} = k_{i\text{в}}\frac{a}{A_6}\left(\frac{a}{A_6}\right)^{\frac{1}{\theta}} L_{i\text{в}}, \tag{34b}$$

$$\bar{\mathrm{P}}_{i\text{в}} = \frac{\bar{\mathrm{P}}_{i\text{в}}}{\mathrm{P}_{i\text{в}6}} = \left(\frac{a}{A_6}\right)^{\frac{1}{\theta}} \frac{\frac{c_Y m}{1+m}a - (r + \delta_{i\text{в}})}{\frac{c_Y m}{1+m}A_6 - (r + \delta_{i\text{в}})} L_{i\text{в}}. \tag{34c}$$

The appearance of the coefficient $k_{i\text{в}}$ in relations (34a) and (34b) is explained by the fact that Arrow's formula (19) has the parameter v, which remains free; i.e., $k_{i\text{в}} = f(v)$. We found the concrete value of the coefficient $k_{i\text{в}}$ by the least squares method proceeding from

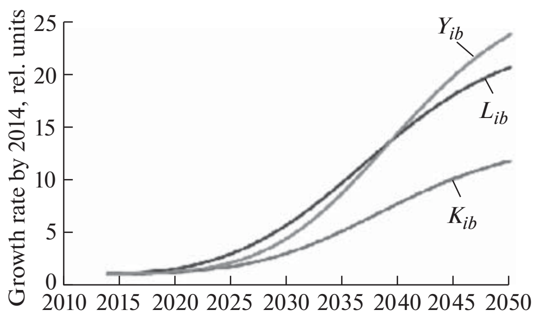

Fig. 3. Dynamics of the basic growth rates of the main economic variables in innovative industries. @ Key: Темп роста к 2014 г. → Growth rate н. ед. → relative units.

the two known values of $Y_{i\text{в}}$: in 2008, $Y_{i\text{в}} = 0.08$ trillion dollars, and in 2015, $Y_{i\text{в}} = 0.4$ trillion dollars [26]. It turned out that $k_{i\text{в}} = 0.183$.

The paths of the basic growth rates of the main economic variables $\overline{Y}_{i\text{в}}$, $\overline{K}_{i\text{в}}$, and $\overline{L}_{i\text{в}}$, calculated by formulas (34b), (34a), and (33), are shown in Fig. 3. As we see from the graphs, they would have grown until the middle of the century and further but for the pivotal point, where the rate of profit stops increasing and begins to drop, signaling to entrepreneurs that the downward trend of the MCC has begun and that it is necessary to switch to labor-saving technologies. As innovative products and services are distributed to markets, demand for labor increases. This growth continues until demand for both labor and respective goods is saturated. In parallel, wages (28c) and production costs increase, which necessitates switching to labor-saving technologies. As a result, labor outflows from new industries, and then wages and overall demand drop, which causes a cutback in the economy with a subsequent recession. This is how the upward trend is replaced by the downward one, which is characterized by a short duration. The key role in this process is played by fluctuations of profit, which act as a switch from a positive conjuncture to a negative one, from the upward trend to the downward one. It is profit that makes it possible to endogenize the MCC mechanism.

MATHEMATICAL MODELS TO DESCRIBE AND CALCULATE THE MCC DOWNWARD TREND

There are two different forces the combined action of which makes the economy decrease at the upper pivotal point. The first one is the effect of technological advance on the rate of profit. When, in the course of the distribution of basic innovative technologies and innovative products, ineffective pseudoinnovations displace enhancing technologies, the productivity capital ratio stops growing and begins to decrease. This leads to a long-term decrease in the rate of profit. This is how conditions for a long overall decline in the economy are created, but its break is accelerated by the second force, the appearance of excessive production capacities and the overaccumulation of fixed capital, which leads to its depreciation. This additional cause worsens the general economic situation. The rate of profit begins to fall increasingly rapidly because capital itself becomes excessive.

Technological advance in innovative industries of the economy. While in the upward trend of the MCC we saw the Poisson flow of enhancing innovations with the intensity $\lambda(t)$ (7), in the downward trend we deal with the flow of pseudoinnovations. The category "pseudoinnovations" was first identified by G. Mensch [9]. As a rule, they are typical of the final stage of the life cycle of a moribund technological mode, when it has practically exhausted its potential but is in every possible way resisting its replacement by a more progressive system and seeking to preserve its niche in the market by some semblance of enhancement. Pseudoinnovations are usually due to force of habit, are conservative and destructive, and, hence, only favor the accelerating depreciation of productive capital and a slowdown in production.

While in the upward trend of the MCC the number of new enterprises increases proportionally to the power of the flow of enhancing innovations, the downward trend, on the contrary, is characterized by bankruptcy of enterprises and their quitting the market. Therefore, we deal with a continuous-time Markov process of the multiplication and death of enterprises, the expected value of which is described by the following Kolmogorov differential equation [27]:

$$\frac{d\sigma}{dt} = \lambda(t) - \chi(t)\sigma(t), \qquad (35)$$

where $\lambda(t)$ is the intensity of the Poisson flow of enhancing innovations leading to the multiplication of enterprises and $\chi(t)$ is the intensity of the flow of pseudoinnovations leading to the death of enterprises. For the upward trend of the MCC, with $T_6 < t \le T_r$ (T_r being the point of transfer from the upward trend to the downward one), we can suppose that $\chi(t) = 0$, Key: remove, and differential equation (35) will acquire the simplest form

$$\frac{d\sigma}{dt} = \lambda(t), \qquad (35a)$$

the solution of which is function (8): $\sigma(t) = \int_0^t \lambda(\tau)d\tau$. As was noted above more than once, to evaluate T_r, it is necessary to use the behavior of the rate of profit in the upward trend. The best for this purpose are the slowdown rates of the rate of profit in the upward trend (30b), which fully coincide with the slowdown rates of labor productivity (see Fig. 2b). For entrepreneurs, there is always a threshold productivity or threshold slowdown rates of the rate of profit below which it is

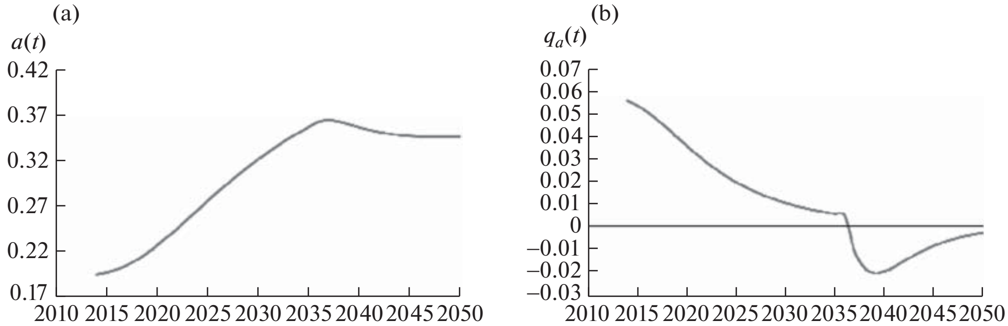

Fig. 4. Cyclical dynamics of (a) the technological level and (b) the rates of labor productivity change in innovative industries of the economy.

not worthwhile for them to invest in the existing industry. This threshold is usually 0.5%. If we take this threshold labor productivity, then it follows from Fig. 3 that $T_r = 2036$ for the chosen value of the parameter $\lambda_0 = 0.25$.

In the downward trend, on the contrary, $\lambda(t)$ can be taken as zero ($\lambda \cong 0$), and then equation (35) is reduced to a simpler form:

$$\frac{d\sigma_r}{dt} = -\chi(t)\sigma_r(t), \quad t > T_r. \tag{35c}$$

The index r means here that $\sigma_r(t)$ relates to the downward MCC trend. Let us take the initial value $\sigma_r(t)$ at $t = T_r$ as approximately $\sigma_m = 1$ at $t \to +\infty$; i.e., $\sigma_r(T_r) = 1$. Naturally, the intensity of pseudoinnovations in the downward trend will be lower than that of enhancing innovations (7) at the initial phase of the upward trend; that is, at the beginning, pseudoinnovations will grow, and then, when their uselessness becomes obvious, the intensity will begin to drop rapidly. This scenario is covered by only one of the numerous Erlang functions (6), namely the Erlang function of the second order:

$$\chi(t) = \lambda_2(t) = \lambda_0^2(t - T_r)e^{-\lambda_0(t-T_r)}. \tag{36}$$

Plugging (36) into (35b) and solving the resultant differential equation, we obtain

$$\sigma_r(t) = \exp\left\{-\left[1 - \lambda_0\left(t - T_r + \frac{1}{\lambda_0}\right)e^{-\lambda_0(t-T_r)}\right]\right\}. \tag{37}$$

The derivative of this function that characterizes the speed of the slowdown of labor productivity has the following form:

$$\dot{\sigma}_r(t) = -\lambda_0^2(t - T_r) \times \exp\left\{[1 + \lambda_0(t - T_r)]\left[e^{-\lambda\cdot(t-T_r)} - 1\right]\right\}. \tag{37a}$$

Consequently, the dynamics of technological advance (16) on the downward wave will be as follows:

$$a_r(t) = A(t) + \varepsilon \cdot \exp\left\{-\left[1 - \lambda_0\left(t - T_r + \frac{1}{\lambda_0}\right)e^{-\lambda_0(t-T_r)}\right]\right\}. \tag{38}$$

$A(t)$ here is still the continuation of logistic function (12), which remains during the entire sixth MCC.

Now we can calculate the slowdown rates of labor productivity in the downward trend of the MCC in innovative industries of the economy:

$$q_{a_r} = \frac{\dot{a}_r}{a_r} = \frac{\dot{A}(t) + \varepsilon\dot{\sigma}_r}{A(t) + \varepsilon\sigma_r}. \tag{38a}$$

Then we calculated graphs for $a_r(t)$ (38) and $q_{ar}(t)$ (38a) in the downward trend of the MCC ($2036 \le t < 2050$) and mated them with the graphs for $a(t)$ (see Fig. 2) and $q_a(t)$ (see Fig. 2b) in the upward trend for the main accepted variant ($\lambda_0^{(1)} = 0.25$). They are shown in Fig. 4. As is clear from Fig. 4a, labor productivity (technological level) in innovative industries increases approximately by two times during the upward trend, while in the downward trend it somewhat decreases, although insignificantly. As for the rates of change in labor productivity (Fig. 4b), there is a smooth decrease during the entire downward trend to 0.5%; then, in transferring to the downward trend, it drops sharply and becomes negative; and then it increases to zero, which testifies to the stagnation of the technological level. This means that it is necessary to seek and introduce new, more productive basic technologies in order to ensure further technological advance.

The dynamics of capital depreciation in innovative industries of the economy. In the downward trend of the MCC, investments in productive capital are made for the sole purpose to amortize its physical depreciation, merely to keep it in operational condition, while pure investments are zero. Obsolescence, which leads to capital depreciation, can be calculated by adding technological advance rates (38a) to the acting capital retirement rate δ (3a), which leads to a quicker obsolescence of the existing industrial equipment [6]. Since technological advance rates in the downward trend of the MCC are negative (see Fig. 4b), they are naturally added with a minus sign. As a result, the classical equation of capital accumulation (30) for new industries of the economy will have the following form:

$$\frac{dK_{i\text{в}}}{dt} = I_K - \left(\delta_{i\text{в}} - n\frac{\dot{a}_r}{a_r}\right)K_{i\text{в}}, \quad t > T_r. \tag{39}$$

Since some of the new industries die during the downward trend as uncompetitive and the survivors become traditional, to determine the value of the parameter n [6], we can use production function (PF) (1), which describes the dynamics of traditional industries. Hence, we can assume $n = \frac{1}{\alpha}$, where $\alpha = 0.48$. As was noted above, during the downward trend, $I_K = 0$. The initial value of $K_{i\text{в}}$ at the point $t = T_r$, which equals the maximal value of $K_{i\text{в}}$, reached by the end of the upward trend, is denoted by $K_{i\text{в}r}$; i.e.,

$$K_{i\text{в}|t=T_r} = K_{i\text{в}r}. \tag{39a}$$

Solving differential equation (39) with account for $I_K = 0$ and $n = \frac{1}{\alpha}$, we obtain

$$\frac{K_{i\text{в}r}}{K_{i\text{в}m}} = \frac{1}{2^n}\left\{1 + \exp\left[\lambda_0\left(t - T_r + \frac{1}{\lambda_0}\right)e^{-\lambda_0\cdot(t-T_r)} - 1\right]\right\}^n. \tag{40}$$

Determining the dynamics of output slowdown and employment decrease in innovative industries. First of all, let us determine the PF type for innovative industries in the downward trend. Since labor-saving technologies are used first and foremost at this stage, it is obviously necessary to use a PF with Harrod-neutral technical progress, i.e.,

$$Y_{i\text{в}r} = K_{i\text{в}r}^{\beta}\left(a_r L_{i\text{в}r}\right)^{1-\beta}. \tag{41}$$

1 The dimensionalities of the variables within this PF are as follows:

$$[Y_{i\text{в}r}] = P, \quad [K_{i\text{в}r}] = PT, \quad [L_{i\text{в}r}] = L, \quad [a_r] = \frac{P}{L}. \tag{41a}$$

1 Since the dimensionalities of all controlling variables $K_{i\text{в}r}$, $L_{i\text{в}r}$, and a_r are independent, we have a fixed dimensionless parameter Π, equal to the constant

$$\Pi = \frac{Y_{i\text{в}r}}{K_{i\text{в}r}^0 a_r L_{i\text{в}r}} = c_{Y_r} = \text{const}. \tag{41b}$$

From this,

$$Y_{i\text{в}r} = c_{Y_r} a_r L_{i\text{в}r}, \tag{41c}$$

i.e., in PF (41), $\beta = 0$.

Now we can write Eq. (26a) for determining real profit:

$$\bar{\text{P}}_{i\text{в}r}(t) = Y_{i\text{в}r} - \bar{w}_r L_{i\text{в}r} - \left(r + \delta_{i\text{в}} - n\frac{\dot{a}_r}{a_r}\right)K_{i\text{в}r}. \tag{42}$$

In this equation, the functional expression $K_{i\text{в}r}(t)$ (40) is already known; the output $Y_{i\text{в}r}(t)$ is expressed through $L_{i\text{в}r}(t)$ (41c). As before (28), we find the optimal level of wages from relation (41c)

$$\bar{w}_r = \frac{\partial Y_{i\text{в}r}}{\partial L_{i\text{в}r}} = c_{Y_r} a_r, \tag{42a}$$

and then the real actual wage rate

$$\bar{w}_r^* = \frac{c_{Y_r} a_r}{1+m}. \tag{42b}$$

Plugging the expressions for $Y_{i\text{в}r}$ (41c) and $\bar{w}_r^*$ (42b) into initial equation (41), we finally obtain

$$\bar{\text{P}}_{i\text{в}r}(t) = \frac{c_{Y_r} a_r m}{1+m}L_{i\text{в}r} - \left(r + \delta_{i\text{в}} - n\frac{\dot{a}}{a_r}\right)K_{i\text{в}r}. \tag{42c}$$

Further, to obtain an equation for determining employment, let us recall that, in the downward trend of the MCC, gross investments $I_{i\text{в}r}$ are proportional to profits $\bar{\text{P}}_{i\text{в}r}$:

$$I_{i\text{в}r} = \dot{K}_{i\text{в}r} + \left(\delta_{i\text{в}} - n\frac{\dot{a}_r}{a_r}\right)K_{i\text{в}r} = \nu_r\left[\frac{c_{Y_r} m a_r}{1+m}L_{i\text{в}r} - \left(r + \delta_{i\text{в}} - n\frac{\dot{a}_r}{a_r}\right)K_{i\text{в}r}\right], \tag{43}$$

and $\nu_r = 0.42$, $a_m = 0.5$, $r = 0.08$, $\delta_{i\text{в}} = 0.15$, and $n = \frac{1}{\alpha}$, where $\alpha = 0.48$ as before.

Since in the downward MCC trend pure investments in fixed capital are zero ($I_{K_{i\text{в}r}} = \dot{K}_{i\text{в}r} + \delta_{i\text{в}} K_{i\text{в}r} = 0$), it follows from (43) that

$$L_{i\text{в}r}(t) = \frac{1+m}{c_{Y_r} m a_r}\left[r + \delta_{i\text{в}} - n\left(1 + \frac{1}{\nu_r}\right)\frac{\dot{a}_r}{a_r}\right]K_{i\text{в}r}(t). \tag{44}$$

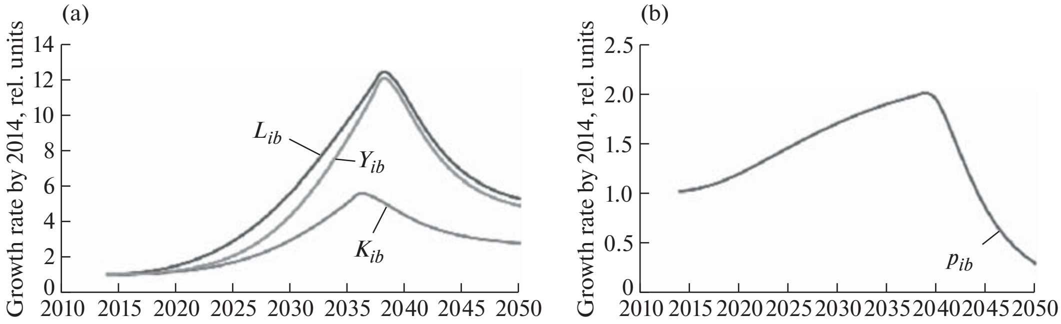

Fig. 5. Cyclical dynamics of the basic growth rates of (a) the main economic variables in innovative industries and the cyclical dynamics of (b) the rate of profit in the sixth MCC.

Taking into account that, in the upper pivotal point, $\dot{a}_r = 0$, for $t = T_r$, we obtain

$$L_{i\text{в}rm} = \frac{1+m}{c_{Y_r} m a_{rm}} (r + \delta_{i\text{в}}) K_{i\text{в}rm}. \quad (44a)$$

Consequently,

$$\bar{L}_{i\text{в}r}(t) = \frac{L_{i\text{в}r}(t)}{L_{i\text{в}rm}} = \frac{a_{rm}}{a_r}\left[1 - \frac{(1+\nu_r)}{\nu_r (r + \delta_{i\text{в}})} \frac{\dot{a}}{a_r}\right] \bar{K}_{i\text{в}r}. \quad (44b)$$

Further, plugging (44) into (41c), we obtain a formula to calculate output in the downward trend of the MCC:

$$Y_{i\text{в}r}(t) = \frac{1+m}{m}\left[r + \delta_{i\text{в}} - n\frac{1+\nu_r}{\nu_r}\frac{\dot{a}_r}{a_r}\right] K_{i\text{в}r}(t). \quad (45)$$

In the form of basic growth rates, this equation will be as follows:

$$\bar{Y}_{i\text{в}r}(t) = \frac{Y_{i\text{в}r}}{Y_{i\text{в}rm}} = \left[1 - \frac{(1+\nu_r)n}{\nu_r (r + \delta_{i\text{в}})} \frac{\dot{a}_r}{a_r}\right] \bar{K}_{i\text{в}r}(t). \quad (45a)$$

With account for (44), from (42c) we obtain final relations to determine and calculate both the profit and the rate of profit:

$$\bar{\mathrm{P}}_{i\text{в}r}(t) = -\frac{n}{\nu_r}\frac{\dot{a}_r}{a_r} K_{i\text{в}r}, \quad (46a)$$

$$\bar{\rho}_{i\text{в}r}(t) = -\frac{n}{\nu_r} q_{a_r}. \quad (46b)$$

It follows from these relations that both profit itself (46a) and its rate (46b) in the downward trend are positive but drop sharply.

Above we obtained relations to calculate all main variables of economic dynamics in the downward trend of the MCC. The graphs $\bar{K}_{i\text{в}r}$, $\bar{L}_{i\text{в}r}$, and $\bar{Y}_{i\text{в}r}$, calculated according to formulas (40), (44b), and (45a), respectively, are shown in Fig. 5, where they are already mated with the respective graphs that describe the dynamics of growth in the upward trend of the MCC. Figure 5b shows a graph of the sharp decline of the rate of profit in the MCC downward trend, calculated according to formula (46b) and mated with the graph of the growth of the rate of profit in the upward trend (30a). The sharp decline of the rate of profit determines the transfer from the MCC upward trend to the downward one, and its fall below the initial level ($\bar{\rho}_{i\text{в}r} < 1$) means the beginning of the depression phase. As we see from this figure, the depression phase will last from 2044 through 2050.

It is under the pressure of the sharp decrease in effectiveness of capital investments, which is quantitatively measured by the rate of profit, when significant capacities have already been accumulated and it is impossible to prevent the economy's fall into the phase of profound long depression, that the most insightful entrepreneurs (Schumpeter's innovators) begin to search for radical innovations based on basic technologies of the future technological mode. Therefore, within the depression phase, the introduction of basic innovations turns out to be the only possibility of profitable investment, and ultimately, as Mensch figuratively puts it [9], innovations overcome depression. The economy as a whole overcomes the lower pivotal point. Further, owing to active measures of numerous imitative entrepreneurs, there occurs a "storm" of enhancing innovations, which forms the upward trend of the next MCC. Thus, the economy evolves endogenously cycle by cycle.

* * *

In conclusion, we present the results of calculating the dynamics of labor productivity (increase in the technological level) across the entire US economy and the paths of its GDP growth, which were obtained on the basis of predictive calculations of similar indicators in innovative industries. Figure 6 shows a graph of the increase in the average technological level $A(t)$ (12) in the US economy during the sixth MCC (2014–2050),

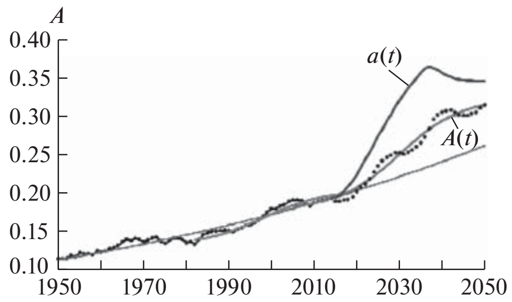

Fig. 6. Forecast dynamics of the growth of the technological level in the US economy for the period of the sixth MCC.

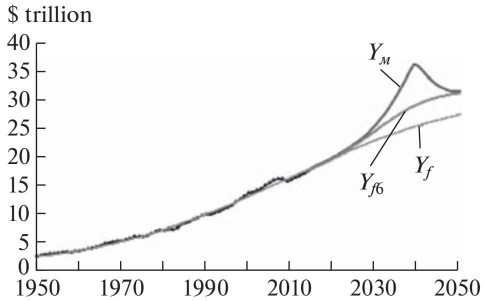

Fig. 7. Forecast trend paths of the movement of the GDP of the US economy for the period of the sixth MCC.

mated with the trend technological level $A_f(t)$ at the beginning of this cycle ($T_в = T_6 = 2014$ г.). Owing to the powerful synergistic effect of NBIC technologies, the forecast technological level in the US economy will turn out to be much higher than the trend. In fact, we will observe cyclic growth of the technological level in compliance with the Poisson law of probability distribution (dotted path), just as was the case in the course of the previous MCCs. Therefore, the coming sixth technological mode will ensure a significant growth of labor productivity both in the entire economy (A) and in innovative industries (a), which promises to bring about huge profits to capitalists to stimulate their investment activity.

Figure 7 shows the trend (Y_f) and forecast (Y_{f6}) paths of the GDP movement for the US economy during the sixth MCC, calculated according to formulas (2) and (1), respectively. Note that, in production function (1) for predictive calculations for the period of the sixth MCC, the trend paths of all the main factors were calculated with account for the cyclic dynamics of the development of the respective variables. We can see from this figure that the forecast path of the GDP movement will run much higher than the secular trend path. The same figure also presents the path of the maximal potentially possible value of the GDP, calculated with respect to the conditions of no resource constraints.

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Translated by B. Alekseev

SPELL: 1. dimensionalities, 2. concretize